Mechanism Design - Problem Set 2

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1. **Optimal Auction with Many Goods.** Consider the following extension of Myerson (1981): there are n buyers whose valuations are independently drawn from a distribution (c.d.f.) $F(v)$, i.e., each is interested in one unit of the good. The seller can sell any number of units $X \in \mathbb{N} \cup \{0\}$ at cost $C(X)$. Suppose that the function $C(\cdot)$ maps $\mathbb{N} \cup \{0\}$ to $\mathbb{R}_+$ and that it is convex, i.e., $C(X+1) - C(X)$ is increasing. Additionally, suppose that $MR(v) = v - \frac{1-F(v)}{f(v)}$ is increasing in v.
   Throughout the exercise, assume that incentive constraints and participation constraints are ex-post, i.e., after each buyer learns other buyers valuations.

   1. Characterize the optimal selling mechanism using direct mechanism. Provide a proof for your reasoning.
   2. Describe an auction format that implements the optimal allocation described in part 1.
   3. Now, suppose that $C(X) = c \cdot X$ for a $c > 0$. Show that optimal allocation can be implemented using a posted price.
   4. Now, suppose that the distribution of buyer's valuations are correlated according to the following structure:

      $$\theta \sim G(\theta), v_i \sim F(v_i; \theta)$$

      with v_i's being independent conditional on θ. The seller does not know θ but the buyers do. Characterize the optimal mechanism. *Hint: Cremer-McLean does not apply here!!*

2. **Dynamic Auctions.** Consider the problem of a seller who is interested in selling two item but can only do this one at a time. Suppose there are n buyers who value only one object equally at v_i but are interested in getting the object early on, i.e., they discount the future at rate β and hence payoffs are given by

   $$v_i x_{i,1} - t_{i,1} + \beta \left(v_i x_{i,2} \mathbf{1}[x_{i,1} = 0] - t_{i,2} \right)$$

   . The values are distributed independently according to c.d.f. $F(v)$. The seller has a discount rate of $\hat{\beta}$.

   1. Define an incentive compatible and feasible auctions.

2. Suppose that the seller can commit to auctions. Formulate the seller's problem and solve for the optimal two-period auction.
3. Would the seller want to revise her auction rules after one round of trading?
4. Now, suppose that the seller cannot committ to auction rules in the second period. Formulate the mechanism design problem. What can you say about optimal auctions in this case?

3. **A Housing Allocation Problem – Alert: This problem is somewhat open-ended! but it is a good problem for a summer paper or a publication in Economics. Given its open-ended nature, you should try to make as much progress as you can by making additional appropriate assumptions! Just like when you write your own papers!!!**
Suppose that the city of Pittsburgh wants to allocate housing to low income families. The city faces various incentive problems: 1. income is not observable or at least it is partially observable. 2. Owners of houses in good neighborhood do not have an incentive to provide low cost housing. In this problem you are to write down a model of this, characterize the set of incentive compatible allocations, and think about the implementation of such a scheme.
To do so, consider a linear model of Pittsburgh, i.e., suppose locations in Pittsburgh are represented with a line of length 1. Suppose that locations are ordered according to their distance to the left-most point on the line given by $q \in [0, 1]$. We want to think about q as the quality of the neighborhood with higher q's being valued higher by all households. Suppose that the density of housing is $F(q)$ with $F(q)$ being the fraction of houses with quality less than q.
There are two sets of agents: owners and renters. Each house on the line is owned by a separate owner. Each renter has a payoff function of $v \times q \times x - m \times t$ where v is how much they value housing of quality q and m is their marginal utility of wealth, i.e., how much they care about prices. Suppose that there is a joint distribution of (m, v) given by $G(\cdot, \cdot)$ being its c.d.f. Suppose that owners have the option of living in their houses and get a value of $w(q)$ where $w(q)$ is a strictly increasing function. Additionally, assume that if a renter does not get allocated a house, they move out of Pittsburgh and receive a value of 0.

1. Describe an allocation of housing for this economy.
2. Suppose that houses were being allocated without any intervention from the government and via competitive market. Define a competitive equilibrium for this economy and describe the resulting allocation.
3. Now suppose that everything is being allocated centrally by a hypothetical planner but incentives for truth-telling and participation need to be respected. Find the set of implementable allocations.
4. Suppose that the government cares only about the renters whose marginal utility of wealth is above a certain threshold. What housing allocations are optimal? How do you implement them?

5. **Optional/very open-ended!** Now, suppose that the government needs to determine which neighborhoods should be included in housing subsidy programs. To do so, they design a signal of the quality of the neighborhood. How should they do this design? What information should they pay attention to?